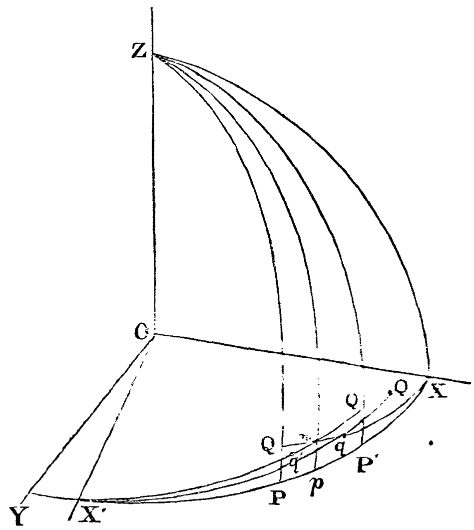
Note on the Curvature of Lines in the Dispersion Spectrum, and the Method of Correcting it. By W. H. M. Christie, Esq.

When a spectrum is formed in the ordinary way with a spectroscope, the lines are curved, and no adjustment of the prisms will get rid of this serious defect. This point has, I believe, not been investigated, though there is no great difficulty in the problem, in the simple form it assumes when the pencil, passing through the prism, consists of parallel rays—a condition which ought always to be satisfied in a properly adjusted spectroscope. It will in this case be sufficient to consider the directions of the rays, in order to show that the image of the slit, after refraction through a prism, will be curved.



Taking the plane of $y\,z$ for the face of the prism on which the rays are incident, O Z for the refracting edge, and O X the normal to the face, let P O, Q O be the directions of pencils from the centre and one extremity of the slit respectively, p O, q O the directions of the corresponding refracted pencils, n the intersection of the arcs Z p and Q X, and let $\angle$ P X Q$=\alpha$,

Then

$$\frac{\sin QX}{\sin qX} = \frac{\sin PX}{\sin pX} = \mu,$$

and

$$\frac{\cot nX}{\cot QX} = \frac{\cos\alpha\cdot\cot pX}{\cos\alpha\cdot\cot PX} = \frac{\cot pX}{\cot PX}$$

by the right-angled spherical triangles P Q X, $p\,q$ X;

$$\therefore \frac{\cos nX}{\cos QX}\cdot\frac{\sin QX}{\sin nX} = \frac{\cos pX}{\cos PX}\cdot\frac{\sin PX}{\sin pX} = \frac{\cos pX}{\cos PX}\cdot\frac{\sin QX}{\sin qX}$$

$$\therefore \sin nX = \frac{\cos PX}{\cos pX}\cdot\frac{\cos nX}{\cos QX}\cdot\sin qX;$$

but

$$\cos nX = \cos pn\cdot\cos pX$$

and

$$\cos QX = \cos PQ\cdot\cos PX$$

$$\therefore \sin nX = \frac{\cos pn}{\cos PQ}\cdot\sin qX > \sin qX,$$

or,

$$nX > qX.$$

Therefore, the image of a straight line after the first refraction is concave towards the normal to the first face. The amount of this curvature may readily be found, as follows:

From the last equation—

$$\frac{\sin nX - \sin qX}{\sin nX + \sin qX} = \frac{\cos pn - \cos PQ}{\cos pn + \cos PQ},$$

or,

$$\frac{\tan \tfrac{1}{2}(nX - qX)}{\tan \tfrac{1}{2}(nX + qX)} = \tan \tfrac{1}{2}(PQ - pn) \cdot \tan \tfrac{1}{2}(PQ + pn),$$

which gives, since $nX - qX$, PQ and pn are small quantities,

$$nX - qX = \tfrac{1}{2}(PQ^2 - pn^2) \cdot \tan qX \text{ nearly;}$$

but

$$\sin PQ = \sin \alpha \sin QX,$$

$$\sin pn = \sin \alpha \sin nX = \sin \alpha \sin qX \text{ nearly}$$

$$\therefore \frac{pn}{PQ} = \frac{\sin qX}{\sin QX} = \frac{1}{\mu};$$

Hence

$$nX - qX = \frac{\mu^2 - 1}{2\mu^2} \cdot PQ^2 \cdot \tan pX \text{ nearly.}$$

For the refraction, on emergence, let OX' be normal to the second face of the prism, $P'O$ the direction of the emergent pencil (incident in the direction PO), $X'nQ'$ an arc of a great circle: then, if we suppose the direction of the emergent pencil reversed, the ray OQ' would be refracted in the direction Oq' (q' being a point on the arc nX'); and therefore the pencil which passes through the prism in the direction Oq will emerge in the direction $OQ_{,}$, $X'qQ_{,}$ being an arc of a great circle, and $Q_{,}X' > Q'X'$; therefore, the image on emergence will be concave towards the normal to the surface of incidence and its curvature, after refraction through the prism, will be—

$$\frac{\mu^2 - 1}{2\mu^2} \cdot PQ^2 (\tan pX + \tan pX'),$$

where pX' may be negative, but is always numerically less than pX, since the deviation is always from the edge of the prism.

This expression will be a minimum when the pencil passes through the prism with minimum deviation, for let $pX = \phi'$ and ι=angle of prism,

$$\tan pX + \tan pX' = \tan \phi' + \tan(\iota - \phi') = \frac{\sin \iota}{\cos \phi' \cos(\iota - \phi')}$$

$$= \frac{2 \sin \iota}{\cos \iota + \cos(2\phi' - \iota)},$$

which is a minimum when $2\phi' = \iota$.

Hence the curvature cannot be got rid of by any adjustment of the prism, and increases with every prism used.

But if, after passing through the train of prisms, the rays be reflected directly back again by a plane mirror, so as to pass through the collimator, and form an image slightly on one side of the slit (which image may be viewed by means of a diagonal prism), the course of the pencil being reversed, the curvature will be corrected by the second passage through the prisms, and the dispersion will be the same as if the pencil had passed through a train, formed by the prisms themselves and their reflections in the plane mirror.

This is done in the Spectroscope now being made for the Royal Observatory, and the lines are perfectly straight.

When the ordinary right-angled prism is used at the end of the train, there is an inversion with regard to up and down which prevents any compensation of this kind from operating.

Blackheath,
1874, *March* 10.

On a Method of drawing, by continued Motion, a very close Approximation to the Parabola, proposed with a view to its possible Application in figuring Reflectors. By F. C. Penrose, Esq.

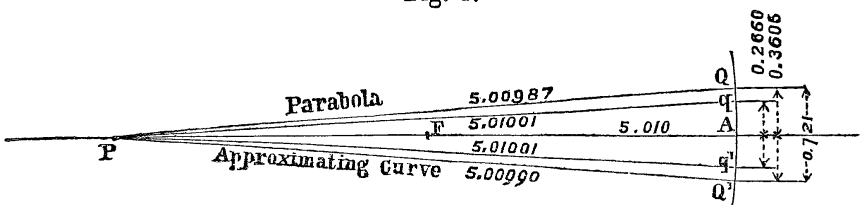
Professor Sylvester, in his Lecture at the Royal Institution in January, showed how two of Peaucellier's parallel motion cells might be combined so as to draw the Conic Sections.

This instrument, however, when arranged so as to draw the Parabola (which it does with theoretical exactness), owing to its working at a mechanical disadvantage, could hardly be made available in practice where steadiness is of the utmost importance.

Another application of the instrument, of which a working model is produced, moves remarkably steadily, and although the curve drawn is not the Parabola in theory—for it is a line of the third order—yet, for a sufficient arc for the majority of optical purposes, may be made to coincide with it so accurately as not to vary from it more than—say, a three hundred-thousandth part of the aperture in a mirror intended for a telescope. I am not speaking of errors of execution, but of theoretic divergence, and various contrivances could be applied to make the execution very exact.

The case which I give in Fig. 1 supposes a reflector of very short focus, not exceeding four times the aperture.

Fig. 1.



The two curves are made to agree at A and at q or q^1, and also to have a common normal P q.

It is true that the circle of curvature at the vertex would agree in measurement at Q with the Parabola as well as this curve does, but the advantage is seen on comparing the intersections of the normals with the axis.

The Parabola is supposed to have for its parameter 10. Consequently, the radius of curvature at the vertex is 5, and all the normals to the circle intersect the axis at that distance from the vertex.

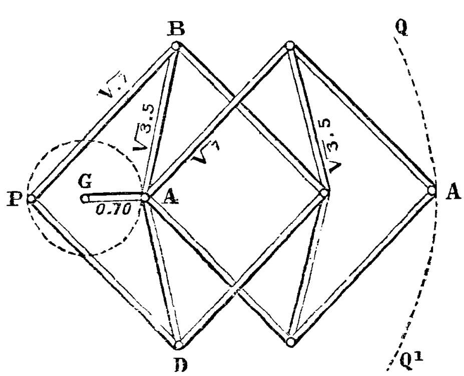
But at q (drawn from an abscissa $= \cdot 01$) there would be just that amount of difference, *viz.*, $\cdot 01$, between the two normals, *i.e.* of the circle and Parabola; but the normal of the proposed curve for this place coincides with that of the Parabola.

Again, at Q, drawn from an abscissa $= \cdot 013$ from the vertex, the parabolic normal intersects the axis at $5 \cdot 013$, but that of the circle is still at $5 \cdot 000$. The proposed curve (of which the normal makes, with the radius vector, an angle of which the tangent is $\frac{\text{N} \sec\theta \tan\theta - \text{M} \sin\theta}{\text{P Q}}$, when N and M are constant ratios, depending upon the adjustments of the instrument) will have its normal intersecting the axis at $5 \cdot 0103$.

If we want a reflector of more curvature, as, for instance, for a lighthouse, we should still be able to secure very great accuracy.

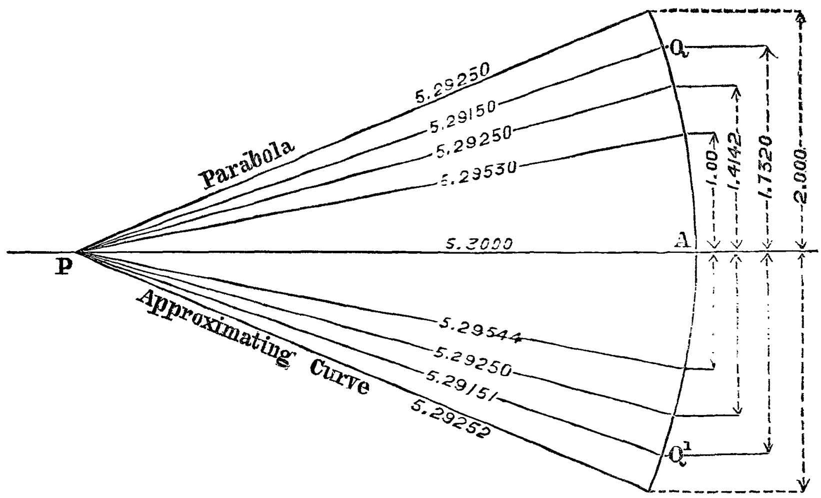
I would propose such a figure as that exhibited in Fig. 3: which would, from the extremities of the reflector, subtend at the focus an angle of about 87°.

Fig. 2.



The instrument arranged for drawing Fig. 3; whilst A traverses in a circle round C, the point Q draws the approximate Parabola.

Fig. 3.



Differences of Radius Vector.

+ ·00008

+ ·00001

± ·0

± ·00014

± ·0

The two curves are put together at the vertex, and also are adjusted at Q or Q^1, where they have a common normal, the radii of curvature having also very small difference.

The abscissæ at the places compared are 0·1, 0·2, 0·3, 0·4.

The polar equation to the curve is $\frac{m+\mu}{m}$ c cos $+\theta\frac{m}{c}$ sec $\theta=\zeta$ where m and μ are the difference of the squares of the lengths of the links forming the cells of the instrument, and c is the diameter of the circle upon which it is made to revolve.

Wimbledon,
1874, *March* 12.

Notes on some Spectroscopic Observations of Sirius, γ Argûs, &c. By E. H. Pringle, Esq.

(*Communicated by Capt. J. Herschel, R.E.*)

The following observations were made at Mangalore, South Canara, with a Spectroscope of two prisms of dense flint glass, with refracting angle of 60°, attached to a $6\frac{1}{2}$-inch silvered-glass speculum telescope.

Most of the observations have been made after removing one

of the prisms, and replacing the short slit by one of greater length, for it was found impossible to keep the image of a star in view with a small field owing to the telescope having altazimuth motion. The measurements, which must only be regarded as approximate, were taken by the angular motion of the small observing telescope.

The eye-piece generally used has a power of about 11.

I first examined *Sirius* on the 11th inst., removing the slit and using a cylindrical lens, and noticed a band in the violet, which I do not find chronicled. Subsequently on the 18th, a night of remarkable clearness, a still more refrangible line was visible, and beyond this again an extent of violet light. On the 19th, and two succeeding nights, after fitting a new slit to the instrument, measurements of these lines were obtained.

A noticeable point in this star's spectrum is the brilliance—comparatively speaking—of the violet light. It argues an absence of the numerous lines and bands which mar this part of the solar spectrum.

The lines X and Y appear very broad, much more so than the hydrogen lines in the star. A cylindrical lens was occasionally used; but for the purpose of measuring I found keeping the observing telescope very slightly out of focus enabled the lines to be distinctly seen, whilst there was less loss of light. Instrumental parallax was guarded against as far as possible, but as before noticed the measurements are only approximate.

γ Argûs. This is the star whose bright line spectrum was observed in December 1871, by Professor Respighi. In January 1872, ignorant of the above discovery, I accidentally hit upon this star, and examined its spectrum with a small direct vision spectroscope, attached to an achromatic of 3-inches aperture.

The three principal bright lines, two in the yellow and one in the blue, were very distinct; and I suspected a fourth still more refrangible than the yellow lines.

I have now re-examined the star, and find the suspected line has an existence. There is, moreover, a fifth line or band near to, and less refrangible than the blue line. This band seems to be defined on the blue, but to fade off gradually towards the red.

I suspect either bright lines or absorption bands in the orange, from the unequal brightness of this part of the spectrum.

Zodiacal Light. The many observations I have taken of this phenomenon have failed to elicit more than a faint diffuse spectrum. I have had—and fear that this season I shall have—no opportunity of examining the Light from the elevation of 6,000 feet, as I had hoped to do.

The Earth-light from the Moon when crescent gives a spectrum terminating abruptly at a point very little less refrangible than E, but fades off gradually in the blue, and is traceable to somewhere about 2250 K.

Mangalore,
1874, *January* 27.